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New developments in the theory of proton radioactivity

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Abstract. Recent advances in the theoretical description of proton emission from spherical and deformed drip line nuclei are reviewed and discussed.

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1 Introduction

The discovery of proton radioactivity allowed to establish [1] the borders of proton stability for nuclei with $50 < Z < 82$ leading to an almost complete identification of the proton drip line in this region of the nuclear chart, and providing important nuclear-structure information on exotic nuclei beyond the drip line. Proton-emitting nuclei with charge above $Z = 68$ have been treated as spherical [2] and the half-lives of the emitted proton calculated within various theoretical approaches showed a quantitative agreement with experimental data. A simple WKB estimation of the transmission through the Coulomb and centrifugal barriers could already suggest the order of magnitude of the decay rates and the angular momentum of the decaying state.

Experimental spectroscopic factors defined as the rate between the calculated and measured half-lives were also determined. From the theoretical point of view, these quantities can be evaluated within the independent quasi-particle BCS approach, where they simply represent the probability that the spherical orbital is empty in the daughter nucleus. A comparison between experimental and theoretical spectroscopic factors for odd-Z and even- or odd-N proton emitters has shown a good correlation between them with the exception of few cases where the experimental value is below the theoretical prediction, indicating a smaller tunnelling probability or fragmentation of the single-particle strength. Some of these nuclei are expected to be deformed; consequently, for them the spherical calculation cannot work and they have to be studied within a different model as will be discussed below. However, a systematic quenching of the measured decay is observed for the emission from the $2d3/2$ in a region where macroscopic-microscopic mass formula calculations practically guarantee a spherical shape. The calculations required to extract the experimental spectroscopic factors rely on the representation of the nuclear mean field by a realistic potential. But the same potential that underestimates the half-lives [2] for the $2d3/2$ level works perfectly for the emission from the $1h11/2$ and $3s1/2$ levels in all spherical nuclei. In ref. [2], the Becchetti-Greenlees [3] potential was used, which has a radius parameter $r_0 = 1.17$ fm. It would be possible to reduce the discrepancy by changing the parameters, as increasing the radius, but it would destroy the consistent picture one wants to achieve. The physical aspect lacking in this approach is the possibility of vibrational excitations.

In this work, we will discuss how it is possible to obtain a unified description of spherical as well as deformed nuclei with the same realistic interaction. The theory for deformed proton emitters will be briefly described in the next section. The last section will be devoted to the effect of particle vibration coupling in proton radioactivity from spherical nuclei, completing the interpretation of all experimental information available at present in this field.

2 Deformed proton emitters

2.1 Odd-Z even-N emitters

The basic assumption in order to study decay from a deformed nucleus is to consider that the decaying nucleon moves in a single-particle Nilsson level, which corresponds to a resonance of the unbound core-proton system. The corresponding single-particle wave function can be found from the solution of the Schrödinger equation

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for a realistic mean field described by a deformed Saxon-Woods potential with a deformed spin-orbit term, imposing outgoing-wave boundary conditions to find the resonance states. The wave function of the parent nucleus is considered as the one of a particle plus rotor in the strong-coupling limit. The decay width for the process is obtained [4] in terms of these wave functions and depends on the escape energy and deformation parameter β. Due to non-conservation of the total angular momentum, the Nilsson levels are quite mixed. For the ground-state decay, only the component of the s.p. wave function with the same angular momentum as the ground state is responsible for the decay, and the width becomes

$$\Gamma_{l_\mathrm{p} j_\mathrm{p}}(r) = \hbar/T = \frac{\hbar^2 k}{\mu(j_\mathrm{p}+1/2)} \frac{|u_{l_\mathrm{p} j_\mathrm{p}}(r)|^2}{|G_{l_\mathrm{p}}(kr) + iF_{l_\mathrm{p}}(kr)|^2}, \quad (1)$$

where F and G are the regular and irregular Coulomb functions, respectively, and $u_{l_\mathrm{p} j_\mathrm{p}}$ the component of the wave function with momentum j_p, equal to the spin of the decaying nucleus. Two effects can be deduced from eq. (1). First, the angular momentum can be smaller than the available spherical s.p. momenta, leading to a shorter half-life. Secondly, the component responsible for the decay can be very small and quite sensitive to calculation details, implying a longer half-life. Therefore, a theoretical width that reproduces the experimental value has clear structure information on the deformation of the decaying nucleus, and properties of the decaying state.

The results of the calculations made with this model [4–7] are shown in table 1 for the known deformed emitters including isomeric decays. The experimental half-lives are perfectly reproduced by a specific state, with defined quantum numbers and deformation, thus leading to an unambiguous assignment of the angular momentum of the decaying states [4,5]. Extra experimental information provided by isomeric decay can also be successfully accounted for by the model. It has been observed in ^{117}La, ^{141}Ho and ^{151}Lu, and the experimental half-lives from the excited states were reproduced in a consistent way with the same deformation that describes ground-state emission.

Table 1. Total angular momentum and deformation that reproduce the experimental half-lives for the measured deformed odd-even proton emitters compared with the predictions of [8]. The theoretical results are from refs. [4–7]. The label "m" refers to decays from isomeric states.

	Proton decay		Möller-Nix	
	J	β	J	β
^{109}I	$1/2^+$	0.14	$1/2^+$	0.16
^{113}Cs	$3/2^+$	$0.15 \div 0.20$	$3/2^+$	0.21
^{117}La	$3/2^+$	$0.20 \div 0.30$	$3/2^+$	0.29
$^{117\mathrm{m}}$La	$9/2^+$	$0.25 \div 0.35$		
^{131}Eu	$3/2^+$	$0.27 \div 0.34$	$3/2^+$	0.33
^{141}Ho	$7/2^-$	$0.30 \div 0.40$	$7/2^-$	0.29
$^{141\mathrm{m}}$Ho	$1/2^+$	$0.30 \div 0.40$		
^{151}Lu	$5/2^-$	$-0.18 \div -0.14$	$5/2^-$	-0.16
$^{151\mathrm{m}}$Lu	$3/2^+$	$-0.18 \div -0.14$		

In deformed nuclei the first-excited rotational state can lie very low in energy. There is a sizeable branching ratio for the decay into such state of the daughter nucleus, known as fine structure and observed in ^{131}Eu, representing a small fraction of the total decay. This data imposes extra constraints to the model that were also consistently fulfilled [6]. It is interesting to observe in all cases agreement with the predictions of Möller-Nix [8].

Recently, a model that takes into account the Coriolis coupling has been presented [9]. However, the formalism was developed in terms of particles instead of quasi-particles, leading to results of doubtful physical meaning [10,11].

2.2 Odd-Z odd-N emitters

The ideas developed in subsect. 2.1 can be generalized to describe emission from deformed systems with an odd number of protons and neutrons. It is expected that the decaying nucleus is in the bandhead state with spin $K_T = |K_\mathrm{p} \pm K_\mathrm{n}|$, otherwise the much faster inband γ transitions would dominate over proton emission. The daughter nucleus will most probably be left in the ground state with $J_\mathrm{d} = |K_\mathrm{n}|$, the magnetic quantum number of the odd neutron, since the neutron intrinsic state does not change during decay and $K_\mathrm{d} = K_\mathrm{n}$. In this way the proton will have the largest available energy. The decaying nucleus is described by a wave function of two-particles–plus–rotor in the strong-coupling limit, represented in terms of the single-particle functions of the odd nucleons, solutions of the Schrödinger equation for a realistic interaction as before. However, in contrast with odd-even nuclei, where the proton is forced to escape with a specific angular momentum, many channels will be open due to the angular-momentum coupling of the proton and daughter nucleus, $\mathbf{J}_\mathrm{d} + \mathbf{j}_\mathrm{p}$, giving a total width for the decay as a sum of partial widths allowed by parity and momentum conservation:

$$\Gamma^{J_\mathrm{d}} = \sum_{j_\mathrm{p}=\max(|J_\mathrm{d}-K_T|,K_\mathrm{p})}^{J_\mathrm{d}+K_T} \Gamma^{J_\mathrm{d}}_{l_\mathrm{p} j_\mathrm{p}}, \quad (2)$$

where the width for the decay in the channel $l_\mathrm{p} j_\mathrm{p}$ is given by

$$\Gamma^{J_\mathrm{d}}_{l_\mathrm{p} j_\mathrm{p}} = \frac{\hbar^2 k}{\mu} \frac{(2J_\mathrm{d}+1)}{(2K_T+1)} \langle J_\mathrm{d}, K_\mathrm{n}, j_\mathrm{p}, K_\mathrm{p} | K_T, K_T \rangle^2 \cdot \frac{|u_{l_\mathrm{p} j_\mathrm{p}}(r)|^2}{|G_{l_\mathrm{p}}(kr) + iF_{l_\mathrm{p}}(kr)|^2} u^2_{K_\mathrm{p}}. \quad (3)$$

The factor $u^2_{K_\mathrm{p}}$ is the probability that the proton single-particle level in the daughter nucleus is empty, evaluated with the pairing residual interaction in the BCS approach. The quantity in brackets indicates a Clebsch-Gordan coefficient resulting from the angular-momentum coupling of the odd nucleons.

Equations (2) and (3) depend on the quantum numbers of the unpaired-neutron state. In this sense, the unpaired neutron is not simply a spectator, but contributes significantly with its angular momentum to the decay.

Table 2. As in table 1 for the measured odd-odd proton emitters. Results from the calculation of ref. [12]. The quantities K_p and K_n are the magnetic quantum numbers of the proton and neutron Nilsson wave functions, J the total angular momentum of the parent nucleus, and β and β_M the deformation coming from the proton decay calculation and the prediction of ref. [8].

	K_p	K_n	J	β	β_M
^{112}Cs	$3/2^+$	$3/2^+$	0^+, 3^+	$0.12 \div 0.22$	0.21
^{140}Ho	$7/2^-$	$9/2^-$, $7/2^+$	8^+, 0^-	$0.26 \div 0.34$	0.30
^{150}Lu	$5/2^-$	$1/2^-$	2^+	$-0.15 \div -0.17$	-0.16
$^{150\mathrm{m}}$Lu	$3/2^+$	$1/2^-$	1^-, 2^-	$-0.22 \div 0.00$	

The results from these calculations [12] are shown in table 2. As for odd-even nuclei, the description of the decay rates was very successful. The same Nilsson state of the odd proton is used in the calculation of odd-odd and neighbour odd-even nuclei as can be seen comparing tables 1 and 2. This represents a further consistency check of the model.

From the present data it is not always possible to identify completely the single-particle configuration of the decaying nucleus, due to the various couplings of the odd nucleons. However, it is still possible to select a configuration that can describe well the data and agrees with predictions made by other models [8], leaving the solution of the remaining ambiguities to future measurements on fine structure and isomeric decays.

It should be noticed that the largest contribution of the residual interaction between the odd neutron and odd proton is the diagonal part, taken into account exactly in our model. The non-diagonal contribution is not very well known, and does not seem to be very important in order to reproduce the experimental data.

The results presented in tables 1 and 2 were obtained with an interaction in terms of the so-called [13] "universal" parameters which has a radius parameter $r_0 = 1.275$ fm. All other calculations in the literature on deformed nuclei [9,10] use similar radius parameters r_0 always greater than 1.24 fm. In all cases r_0 is much larger than the one of the Becchetti-Greenlees potential used to describe spherical nuclei as discussed in sect. 1. The physical origin of this discrepancy will be discussed in the next section.

3 Spherical proton emitters and the particle vibration coupling

Spherical nuclei have the possibility of collective excitations and display a vibrational spectrum with some unharmonicity. It is reasonable to expect a correlation between the outgoing proton and the lowest 2^+ excited state of the daughter nucleus. The spectroscopic factor should be very sensitive to this coupling. The observation of fine structure in proton decay from ^{145}Tm [14] confirms this hypothesis. The emitted protons had the same half-life, and left the daughter nucleus ^{144}Eu in the ground and first 2^+ excited state.

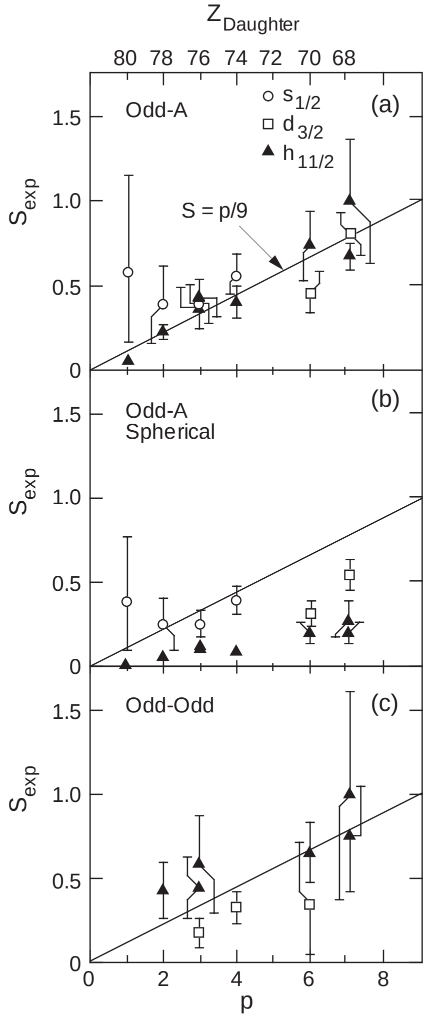

Fig. 1. (a) Experimental spectroscopic factors calculated with particle vibration coupling for odd-A emitters as a function of the number of pairs of p-holes in the daughter nucleus, below $Z = 82$. (b) As in (a) without particle vibration coupling. (c) As in (a) for odd-odd proton emitters. Figure from ref. [15].

In a spherical system, the odd proton is considered to be coupled only to the 0^+ ground state of the daughter nucleus, thus preventing these decays. This could be possible in a deformed system, but agreement with the experimental half-life and branching ratio was found [15] only for an oblate $K = \frac{5}{2}^-$ ground state not compatible with other theoretical predictions. Alternatively, the nucleus can be considered as having a time-averaged spherical shape, but the wave function contains components due to particle vibration coupling. The proton will take a longer time to escape, since the probability of leaving the daughter nucleus in the ground state is smaller than 1. Therefore, decays that were well described within the spherical approach will require an interaction with a larger radius to compensate the smaller spectroscopic factor.

The role of core excitations has been studied in a coupled-channel approach [15]. The intrinsic vibrational Hamiltonian of the daughter nucleus is added to the single-particle mean field plus the coupling between both terms. Up to first order in the vibrational amplitudes $\alpha_{\lambda\mu}$, the vibrational coupling has the general form

$$V_{\rm vibc}(r, \alpha_{\lambda\mu}) = \left[-R \frac{{\rm d}V(r)}{{\rm d}r} \right] \sum_{\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\mathbf{r}) , \qquad (4)$$

where R is the radius of the nuclear and Coulomb central interaction V. The quadrupole amplitudes $\alpha_{2\mu}$ can be determined directly from the excitation energy of the 2^+ state in the daughter nucleus, therefore it is not an extra parameter. For these interactions, the Schrödinger equation reduces to a set of coupled-channel equations that, combined with the distorted wave Green's function method to account for the long-range Coulomb quadrupole interaction, determine the proton decay rates. For details of the calculation see ref. [15].

In fig. 1(a) and (c) the theoretical and experimental spectroscopic factors of spherical odd-even and odd-odd nuclei are compared, showing a perfect agreement. The deviations previously observed [2] in a pure spherical calculations for the $2d3/2$ proton orbital were eliminated. Furthermore, the interaction used, was the same that describes the decay rates of deformed emitters [10]. The experimental spectroscopic factors calculated in a pure spherical picture with this potential are displayed in fig. 1(b). The agreement with most part of the theoretical cases is destroyed, since the radius used is larger than the one of the Becchetti-Greenlees potential used in ref. [2]. A similar particle vibration coupling calculation [16] with the Becchetti-Greenlees potential, needs as expected, quite large deformations in order to reproduce the data. However, the vibrational model is inconsistent with such deformations.

The results discussed in this section clearly show the importance of particle vibration coupling in the decay of quasi-spherical emitters, and solves the puzzle of the need of different radii going from spherical to deformed systems.

4 Conclusions

A complete and unified understanding of proton decay in spherical and deformed nuclei is by now achieved. Open questions in the comparison of the experimental and theoretical spectroscopic factors for quasi-spherical nuclei, were solved by the inclusion of a vibrational coupling to the first-excited state of the daughter nucleus. A model was developed to describe decay from odd-even and generalized to odd-odd nuclei describing very well all available experimental results and also supporting previous predictions made by other models on nuclear-structure properties of the decaying nucleus.

New tools have been developed to the analysis of future data, and to give important contributions to explore the structure of exotic nuclei in the region of the proton drip line.

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References

1. P.J. Woods, C.N. Davids, Annu. Rev. Nucl. Part. Sci. **47**, 541 (1997).
2. S. Åberg, P.B. Semmes, W. Nazarewicz, Phys. Rev. C **56**, 1762 (1997).
3. F.D. Becchetti, G.W. Greenlees, Phys. Rev. **182**, 1190 (1969).
4. E. Maglione, L.S. Ferreira, R.J. Liotta, Phys. Rev. Lett. **81**, 538 (1998); Phys. Rev. C **59**, R589 (1999).
5. L.S. Ferreira, E. Maglione, Phys. Rev. C **61**, 021304(R) (2000).
6. E. Maglione, L.S. Ferreira, Phys. Rev. C **61**, 47307 (2000).
7. F. Soramel *et al.*, Phys. Rev. C **63**, 031304(R) (2001).
8. P. Möller, J.R. Nix, W.D. Myers, W.J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995); P. Möller, R.J. Nix, K.L. Kratz, At. Data Nucl. Data Tables **66**, 131 (1997).
9. A.T. Kruppa, B. Barmore, W. Nazarewicz, T. Vertse, Phys. Rev. Lett. **84**, 4549 (2000); B. Barmore, A.T. Kruppa, W. Nazarewicz, T. Vertse, Phys. Rev. C **62**, 054315 (2000).
10. H. Esbensen, C.N. Davids, Phys. Rev. C **63**, 014315 (2001).
11. G. Fiorin, E. Maglione, L.S. Ferreira, in preparation.
12. L.S. Ferreira, E. Maglione, Phys. Rev. Lett. **86**, 1721 (2001).
13. S. Cwiok, J. Dudek, W. Nazarewicz, J. Skalski, T. Werner, Comp. Phys. Comm. **46**, 379 (1987).
14. K.P. Rykaczewski *et al.*, Nucl. Phys. A **682**, 270c (2001).
15. Cary N. Davids, Henning Esbensen, Phys. Rev. C **64**, 034317 (2001).
16. K. Hagino, Phys. Rev. C **64**, 041304 (2001).